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$B^0 \to \phi\phi$ decay in perturbative QCD approach

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Abstract. The rare decay $B^0 \to \phi\phi$ can occur only via the penguin annihilation topology in the standard model. We calculate this channel in the perturbative QCD approach. The predicted branching ratio is very small: around 10^{-8}. We also give the polarization fractions, which show that the transverse polarization contribution is comparable to the longitudinal one, due to a big transverse contribution from factorizable diagrams. The small branching ratio in SM makes it sensitive to any new physics contributions.

1 Introduction

The study of B meson decays has proven to be a good place to test the standard model (SM) and to give some important constraints on the SM parameters. Recently, more attention has been paid to the $B \to VV$ decay modes. The transverse polarization of the vector meson can contribute to the decay width, and the fraction of each kind of polarization has been or will be measured. In some penguin dominated decay modes, such as $B \to \phi K^*$ [1], the experimental results for polarization differ from most theoretical predictions [2], which have been considered as a puzzle and there has been much discussion [3, 4]. So the polarization problem in the $B \to VV$ decay modes brings a new challenge to the standard model; maybe it is a signal of new physics [4, 5].

In this work we will calculate the branching ratio and the polarization fractions of the charmless decay channel $B^0 \to \phi\phi$ with the perturbative QCD approach (PQCD) [6, 7]. In this channel, the initial $\bar{b}$ quark and the light valence d quark in the B meson do not appear in the final states, so we must have the annihilation topology in Feynman diagrams. Annihilation diagrams cannot be calculated in the factorization approach [8, 9] or in the QCD improved factorization approach [10] for its endpoint singularity, but in the PQCD approach this singularity can be regulated by a Sudakov form factor and threshold resummation, so the PQCD calculations can give converging results and have predictive power. In this channel, since no tree level operators can contribute, the dominant contribution comes from penguin operators. The annihilation topology is usually suppressed relative to the emission topology which can appear in other modes, so this channel is a rare decay mode and has not been measured in the B factories.

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In the next section we give our theoretical formulae based on the PQCD framework. Then we show the numerical results and a brief conclusion in the third section.

2 Perturbative calculation

For simplicity, we work in the B meson rest frame and adopt the light-cone coordinate system. Then the four-momentum of the B meson and the two ϕ mesons in the final state can be written as

$$P_1 = \frac{M_B}{\sqrt{2}}\left(1, 1, \mathbf{0}_\mathrm{T}\right),$$
$$P_2 = \frac{M_B}{\sqrt{2}}\left(1-r, r, \mathbf{0}_\mathrm{T}\right),$$
$$P_3 = \frac{M_B}{\sqrt{2}}\left(r, 1-r, \mathbf{0}_\mathrm{T}\right), \qquad (1)$$

in which r is defined by $r = \frac{1}{2}\left(1 - \sqrt{1 - 4M_\phi^2/M_B^2}\right) \simeq M_\phi^2/M_B^2 \ll 1$. To extract the helicity amplitudes, we should parameterize the polarization vectors. The longitudinal polarization vector must satisfy the orthogonality and normalization: $\epsilon_{2\mathrm{L}} \cdot P_2 = 0$, $\epsilon_{3\mathrm{L}} \cdot P_3 = 0$, and ${\epsilon_{2\mathrm{L}}}^2 = {\epsilon_{3\mathrm{L}}}^2 = -1$. Then we can give the manifest form as follows:

$$\epsilon_{2\mathrm{L}} = \frac{1}{\sqrt{2r}}\left(1-r, -r, \mathbf{0}_\mathrm{T}\right),$$
$$\epsilon_{3\mathrm{L}} = \frac{1}{\sqrt{2r}}\left(-r, 1-r, \mathbf{0}_\mathrm{T}\right). \qquad (2)$$

As to the transverse polarization vectors, we can choose the simple form:

$$\epsilon_{2\mathrm{T}} = \frac{1}{\sqrt{2}}\left(0, 0, \mathbf{1}_\mathrm{T}\right),$$

$$\epsilon_{3\mathrm{T}} = \frac{1}{\sqrt{2}}\,(0,0,\mathbf{1}_{\mathrm{T}})\,. \tag{3}$$

Only penguin operators can contribute to this decay channel, so the relevant effective weak Hamiltonian can be written as [11]

$$\mathcal{H}_{\mathrm{eff}} = \frac{G_{\mathrm{F}}}{\sqrt{2}} V_{tb}V_{td}^{*} C_i(\mu) O_i(\mu), \quad i = 3\text{– }10, \tag{4}$$

where C_i are the QCD corrected Wilson coefficients, and O_i are the usual penguin operators with the form

$$\begin{aligned}
O_3 &= (\bar{s}_i b_i)_{V-A} \sum_q (\bar{q}_j q_j)_{V-A}\,,\\
O_4 &= (\bar{s}_i b_j)_{V-A} \sum_q (\bar{q}_j q_i)_{V-A}\,,\\
O_5 &= (\bar{s}_i b_i)_{V-A} \sum_q (\bar{q}_j q_j)_{V+A}\,,\\
O_6 &= (\bar{s}_i b_j)_{V-A} \sum_q (\bar{q}_j q_i)_{V-A}\,,\\
O_7 &= \frac{3}{2}\,(\bar{s}_i b_i)_{V-A} \sum_q e_q\,(\bar{q}_j q_j)_{V+A}\,,\\
O_8 &= \frac{3}{2}\,(\bar{s}_i b_j)_{V-A} \sum_q e_q\,(\bar{q}_j q_i)_{V+A}\,,\\
O_9 &= \frac{3}{2}\,(\bar{s}_i b_i)_{V-A} \sum_q e_q\,(\bar{q}_j q_j)_{V-A}\,,\\
O_{10} &= \frac{3}{2}\,(\bar{s}_i b_j)_{V-A} \sum_q e_q\,(\bar{q}_j q_i)_{V-A}\,,
\end{aligned} \tag{5}$$

where $q = s$. The first four operators are QCD penguin operators, while the last four ones are electroweak penguin operators, which should be suppressed by the coupling $\alpha/\alpha_{\mathrm{s}}$.

The decay width for this channel is

$$\Gamma = \frac{1}{2}\,\frac{G_{\mathrm{F}}^2 |P_c|}{16\pi M_B^2} |V_{tb}^{*}V_{td}|^2 \sum_{\sigma=\mathrm{L,T}} \mathcal{M}^{\sigma\dagger}\mathcal{M}^{\sigma}, \tag{6}$$

where $\boldsymbol{P}_c$ is the three-momentum of the final state meson, with $|\boldsymbol{P}_c| = \frac{M_B}{2}(1-2r)$. Note that for our case an additional factor 1/2 should appear for the permutation symmetry of the identical final state particles. The decay amplitude $\mathcal{M}^{\sigma}$ which is decided by QCD dynamics will be calculated later in the PQCD approach. The subscript σ denotes the helicity states of the two vector mesons with L(T) standing for the longitudinal (transverse) components. After analyzing the Lorentz structure, the amplitude can be decomposed into [1]

$$\begin{aligned}
\mathcal{M}^{\sigma} &= M_B^2 \mathcal{M}_{\mathrm{L}} + M_B^2 \mathcal{M}_{\mathrm{N}} \epsilon_2^{*}(\sigma = \mathrm{T}) \cdot \epsilon_3^{*}(\sigma = \mathrm{T})\\
&\quad + \mathrm{i}\mathcal{M}_{\mathrm{T}} \epsilon_{\mu\nu\rho\sigma} \epsilon_2^{\mu *} \epsilon_3^{\nu *} P_2^{\rho} P_3^{\sigma}.
\end{aligned} \tag{7}$$

We can define the longitudinal H_0, and the transverse $H_{\pm}$ helicity amplitudes as

$$H_0 = M_B^2 \mathcal{M}_{\mathrm{L}},\;\; H_{\pm} = M_B^2 \mathcal{M}_{\mathrm{N}} \mp M_{\phi}^2 \sqrt{r'^2 - 1}\mathcal{M}_{\mathrm{T}}, \tag{8}$$

where $r' = (P_2 \cdot P_3)/M_{\phi}^2$. After the helicity summation, we can deduce that they satisfy the relation

$$\sum_{\sigma=\mathrm{L,R}} \mathcal{M}^{\sigma\dagger}\mathcal{M}^{\sigma} = |H_0|^2 + |H_+|^2 + |H_-|^2. \tag{9}$$

There is another equivalent set of definitions of helicity amplitudes:

$$\begin{aligned}
A_0 &= -\xi M_B^2 \mathcal{M}_{\mathrm{L}},\\
A_{\parallel} &= \xi\sqrt{2} M_B^2 \mathcal{M}_{\mathrm{N}},\\
A_{\perp} &= \xi M_{\phi}^2 \sqrt{r'^2 - 1}\mathcal{M}_{\mathrm{T}},
\end{aligned} \tag{10}$$

with ξ the normalization factor satisfying

$$|A_0|^2 + |A_{\parallel}|^2 + |A_{\perp}|^2 = 1, \tag{11}$$

where the notations A_0, $A_{\parallel}$, $A_{\perp}$ denote the longitudinal, parallel, and perpendicular polarization amplitude.

What follows is a calculation of the matrix elements $\mathcal{M}_{\mathrm{L}}$, $\mathcal{M}_{\mathrm{N}}$ and $\mathcal{M}_{\mathrm{T}}$ of various operators in the weak Hamiltonian with PQCD approach. In the PQCD approach, the decay amplitude is factorized into the convolution of the mesons' light-cone wave functions, the hard scattering kernel and the Wilson coefficients, which stands for the soft, hard and harder dynamics, respectively. The transverse momentum was introduced so that the endpoint singularity which will break the collinear factorization is regulated and the large double logarithm term appears after the integration on the transverse momentum, which is then resummed into the Sudakov form factor. The formalism can be written as

$$\begin{aligned}
\mathcal{M} \sim \int &\mathrm{d}x_1 \mathrm{d}x_2 \mathrm{d}x_3 b_1 \mathrm{d}b_1 b_2 \mathrm{d}b_2 b_3 \mathrm{d}b_3\\
&\times \mathrm{Tr}\big[C(t) \varPhi_B(x_1, b_1) \varPhi_{\phi}(x_2, b_2) \varPhi_{\phi}(x_3, b_3)\\
&\times H(x_i, b_i, t) S_t(x_i) \mathrm{e}^{-S(t)}\big],
\end{aligned} \tag{12}$$

where b_i is the conjugate space coordinate of the transverse momentum, which represents the transverse interval of the meson. t is the largest energy scale in the hard function H, while the jet function $S_t(x_i)$ comes from the summation of the double logarithms $\ln^2 x_i$, called a threshold resummation [12], which becomes large near the endpoint.

The light-cone wave functions of mesons are not calculable in principle in PQCD, but they are universal for all the decay channels. Thus they can be constrained from the measured other decay channels, like $B \to K\pi$ and $B \to \pi\pi$ decays etc. [7]. For the heavy B meson, we have

$$\frac{1}{\sqrt{2N_c}}\,(\not{P}_1 + M_B)\gamma_5 \phi_B(x, b). \tag{13}$$

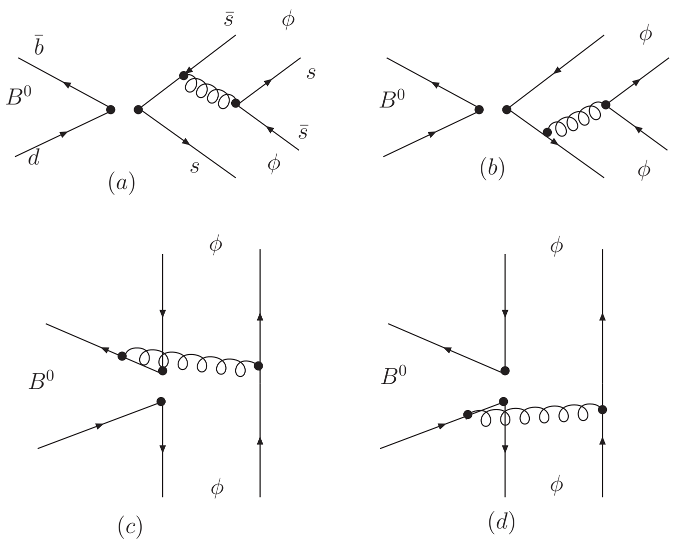

Fig. 1a–d. Leading order Feynman diagrams for $B^0 \to \phi\phi$

For the longitudinal polarized ϕ meson,

$$\frac{1}{\sqrt{2N_c}}\left[M_\phi \not\epsilon_{2\mathrm{L}}\phi_\phi(x)+ \not\epsilon_{2\mathrm{L}} \not P_2\phi^t_\phi(x) + M_\phi I\phi^s_\phi(x)\right], \tag{14}$$

and for transverse polarized ϕ meson,

$$\frac{1}{\sqrt{2N_c}}\left[M_\phi \not\epsilon_{2\mathrm{T}}\phi^v_\phi(x)+ \not\epsilon_{2\mathrm{T}} \not P_2\phi^{\mathrm{T}}_\phi(x) + \frac{M_\phi}{P_2\cdot n_-}\mathrm{i}\epsilon_{\mu\nu\rho\sigma}\gamma_5\gamma^\mu\epsilon^\nu_{2\mathrm{T}}P_2^\rho n_-^\sigma\phi^a_\phi(x)\right]. \tag{15}$$

In the following, we omit the subscript of the ϕ meson for simplicity.

Now the only thing left is the hard part H. In the PQCD approach, it contains the corresponding four quark operator and the hard gluon connecting the quark pair from the sea. They altogether make an effective six quark interaction. The hard part H is channel dependent, but it is perturbatively calculable. When calculating the hard parts (shown in the Fig. 1), the factorizable diagrams (a) and (b) have strong cancellation effects, which results in a null longitudinal polarization contribution and null parallel polarization contribution. The perpendicular polarization survives with a large factorizable contribution, which will be shown later to make a large transverse polarization. The detailed formulas with polarization $\mathcal{M}_{\mathrm{L}}$, $\mathcal{M}_{\mathrm{N}}$, and $\mathcal{M}_{\mathrm{T}}$ for each diagram are given in the appendix. According to PQCD power counting rules, the longitudinal non-factorizable diagram should give the leading contribution, and the contributions from the other diagrams are suppressed by a factor r.

3 Numerical results and summary

For the B meson wave function distribution amplitude in (13), we employ the model [7]

$$\phi_B(x)=N_B x^2(1-x)^2\exp\left[-\frac{1}{2}\left(\frac{xM_B}{\omega_B}\right)^2-\frac{\omega_B^2 b^2}{2}\right], \tag{16}$$

where the shape parameter $\omega_B = 0.4\,\mathrm{GeV}$ has been constrained in other decay modes. The normalization constant $N_B = 91.784\,\mathrm{GeV}$ is related to the B decay constant $f_B = 0.19\,\mathrm{GeV}$. It is one of the two leading twist B meson wave functions; the other one is power suppressed, so we omit its contribution in the leading power analysis [13]. The ϕ meson distribution amplitude up to twist-3 are given by [14]

$$\phi_\phi(x) = \frac{3f_\phi}{\sqrt{2N_c}}x(1-x), \tag{17}$$

$$\phi^t_\phi(x) = \frac{f^{\mathrm{T}}_\phi}{2\sqrt{2N_c}} \times\left\{3(1-2x)^2+1.68C_4^{\frac{1}{2}}(1-2x) +0.69\left[1+(1-2x)\ln\frac{x}{1-x}\right]\right\}, \tag{18}$$

$$\phi^s_\phi(x) = \frac{f^{\mathrm{T}}_\phi}{4\sqrt{2N_c}} \times\left[3(1-2x)\left(4.5-11.2x+11.2x^2\right) +1.38\ln\frac{x}{1-x}\right], \tag{19}$$

$$\phi^{\mathrm{T}}_\phi(x) = \frac{3f^{\mathrm{T}}_\phi}{2\sqrt{2N_c}}x(1-x)\left[1+0.2C_4^{\frac{3}{2}}(1-2x)\right], \tag{20}$$

$$\phi^v_\phi(x) = \frac{f^{\mathrm{T}}_\phi}{2\sqrt{2N_c}} \times\left\{\frac{3}{4}\left[1+(1-2x)^2\right]+0.24\left[3(1-2x)^2-1\right] +0.96C_4^{\frac{1}{2}}(1-2x)\right\}, \tag{21}$$

$$\phi^a_\phi(x) = \frac{3f^{\mathrm{T}}_\phi}{4\sqrt{2N_c}}(1-2x) \times\left[1+0.93(10x^2-10x+1)\right], \tag{22}$$

with the Gegenbauer polynomials

$$C_2^{\frac{1}{2}}(\xi) = \frac{1}{2}\left(3\xi^2-1\right), \tag{23}$$

$$C_4^{\frac{1}{2}}(\xi) = \frac{1}{8}\left(35\xi^4-30\xi^2+3\right), \tag{24}$$

$$C_2^{\frac{3}{2}}(\xi) = \frac{3}{2}\left(5\xi^2-1\right). \tag{25}$$

We employ the following constants [15]: the Fermi coupling constant $G_{\rm F} = 1.16639 \times 10^{-5}\,{\rm GeV}^{-2}$, the CKM matrix element $|V_{tb}^* V_{td}| = 0.0084$, the meson masses $M_B = 5.28\,{\rm GeV}$, $M_\phi = 1.02\,{\rm GeV}$, the decay constants $f_\phi = 0.237\,{\rm GeV}$, $f_\phi^{\rm T} = 0.22\,{\rm GeV}$ and the B meson lifetime $\tau_{B^0} = 1.55\,{\rm ps}$. The results for the center value of the branching ratio is then

$$\mathrm{Br}\left(B^0 \to \phi\phi\right) = 1.89 \times 10^{-8}, \tag{26}$$

and the helicity amplitudes are given by

$$R_0 = 0.65,\ R_\parallel = 0.02,\ R_\perp = 0.33, \tag{27}$$

which shows that the transverse polarization contribution is comparable to the longitudinal one. The relative strong phases $\phi_\parallel = \arg\left(-A_\parallel/A_0\right)$, $\phi_\perp = \arg\left(-A_\perp/A_0\right)$ are given by

$$\phi_\parallel = 198.34^\circ,\ \phi_\perp = 195.48^\circ. \tag{28}$$

Now we consider the contribution from different operators. In the factorizable diagrams, $\mathcal{M}_{\rm L} = \mathcal{M}_{\rm N} = 0$, because of the cancellation between the diagrams of Fig. 1a,b. For $\mathcal{M}_{\rm T}$, the QCD penguin operators O_3, O_4, O_5 and O_6, contribute at the same level. In the non-factorizable diagrams, the operator O_6 gives the most important contributions. If we omit the contribution from the electroweak penguin operators, the variation of the contribution from non-factorizable diagrams (Fig. 1c,d) is small, while that of the factorizable diagrams (Fig. 1a,b) is large. The reason is that the electroweak penguin operator O_9, which has a large Wilson coefficient, only is present in the factorizable diagrams. The overall contribution of the electroweak penguin at the branching ratio level is less than 30%. We also test the contribution without twist-3 wave functions. We find that if we keep only twist-2 wave functions, the total branching ratio does not change much, but the contribution from the factorizable diagrams will vanish, and the transverse polarization contribution then becomes very small. So the twist-3 wave functions give very important corrections to the polarization fractions.

There are many theoretical uncertainties in the calculation. The next to leading order corrections to the hard part is a very important kind of uncertainty for penguin dominant decays. To test it, we consider the hard scale at the range

$$\max\left(0.75 M_B D_a, \frac{1}{b_2}, \frac{1}{b_3}\right) < t_a < \max\left(1.25 M_B D_a, \frac{1}{b_2}, \frac{1}{b_3}\right), \tag{29}$$

$$\max\left(0.75 M_B D_b, \frac{1}{b_2}, \frac{1}{b_3}\right) < t_b < \max\left(1.25 M_B D_b, \frac{1}{b_2}, \frac{1}{b_3}\right), \tag{30}$$

$$\max\left(0.75 M_B F, 0.75 M_B D_c, \frac{1}{b_1}, \frac{1}{b_3}\right) < t_c < \max\left(1.25 M_B F, 1.25 M_B D_c, \frac{1}{b_1}, \frac{1}{b_3}\right), \tag{31}$$

$$\max\left(0.75 M_B F, 0.75 M_B |X|^{\frac{1}{2}}, \frac{1}{b_1}, \frac{1}{b_3}\right) < t_d < \max\left(1.25 M_B F, 1.25 M_B |X|^{\frac{1}{2}}, \frac{1}{b_1}, \frac{1}{b_3}\right), \tag{32}$$

and the other parameters are fixed. Then we can obtain the value of the branching ratio ranging in

$$\mathrm{Br}\left(B^0 \to \phi\phi\right) = \left(1.89^{+0.61}_{-0.21}\right) \times 10^{-8}, \tag{33}$$

which is sensitive to the change of t, so the next to leading order corrections will give an important contribution. The ratios $|A_0|^2$, $|R_\parallel|^2$ and $|R_\perp|^2$ are also very sensitive to the variation of t, because the non-factorized contributions decrease with increasing t, but the factorizable diagrams, which give the main contribution of the transverse polarization, increase. The variation range of $|A_0|^2$ is about 0.41–0.81.

Another uncertainty is from the meson wave functions, which is governed by other measured decays [7]. The variation of the parameters will also give corrections, such as the parameter ω_b in the B wave function; if we assume that its value range is 0.32–0.48, we can give the branching ratio:

$$\mathrm{Br}\left(B^0 \to \phi\phi\right) = \left(1.89^{+0.28}_{-0.26}\right) \times 10^{-8}. \tag{34}$$

The ratios $R_0, R_\parallel, R_\perp$ are not very sensitive to the change of ω_b, because it only gives an overall change of the branching ratio, not to the individual polarization amplitudes.

In this paper, we calculate the rare decay channel $B^0 \to \phi\phi$ in the PQCD approach and give its branching ratio and polarization fractions in the SM. This decay occurs purely via the annihilation topology, and only penguin operators can contribute. We predict that it has a very small branching ratio, 10^{-8}. This is so small that it will be sensitive to new physics, such as supersymmetry etc. [4, 16], which may give a larger branching ratio. The current experiments only give the upper limit: $\mathrm{Br}(B^0 \to \phi\phi) < 1.5 \times 10^{-6}$ [17], so more accurate experimental results are needed to test the theory.

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Appendix A: Factorization formulas

In the factorizable diagrams, due to the identical particles at the final states, cancellation occurs between the two diagrams of Fig. 1a,b. Only the perpendicular polarization part survives,

$$\mathcal{M}_{\mathrm{T}}^{a} = -16\pi C_F f_B M_B^2 \times \int_0^1 \mathrm{d}x_2 \mathrm{d}x_3 \int_0^\infty b_2 \mathrm{d}b_2 \int_0^\infty b_3 \mathrm{d}b_3 r \alpha_{\mathrm{s}}(t) \times [\phi^v(x_2)\phi^v(x_3)(x_3-1) + \phi^v(x_2)\phi^a(x_3)(1+x_3) + \phi^a(x_2)\phi^v(x_3)(1+x_3) + \phi^a(x_2)\phi^a(x_3)(x_3-1)] \times S_\phi(t)^2 \left[C_3 + \frac{C_4}{3} - C_5 - \frac{C_6}{3} + \frac{1}{2}\left(C_7 + \frac{C_8}{3}\right) - \frac{1}{2}\left(C_9 + \frac{C_{10}}{3}\right)\right](t) h(x_2, x_3, b_2, b_3), \quad (A.1)$$

$$\mathcal{M}_{\mathrm{T}}^{b} = 16\pi C_F f_B M_B^2 \times \int_0^1 \mathrm{d}x_2 \mathrm{d}x_3 \int_0^\infty b_2 \mathrm{d}b_2 \int_0^\infty b_3 \mathrm{d}b_3 r \alpha_{\mathrm{s}}(t) \times [\phi^v(x_2)\phi^v(x_3)(x_2) + \phi^v(x_2)\phi^a(x_3)(2-x_2) + \phi^a(x_2)\phi^v(x_3)(2-x_2) + \phi^a(x_2)\phi^a(x_3)(x_2)] \times S_\phi(t)^2 \left[C_3 + \frac{C_4}{3} - C_5 - \frac{C_6}{3} + \frac{1}{2}\left(C_7 + \frac{C_8}{3}\right) - \frac{1}{2}\left(C_9 + \frac{C_{10}}{3}\right)\right](t) h'(x_2 x_3, b_2, b_3), \quad (A.2)$$

where the h functions come from the integral on the transverse momentum, and their manifest form is

$$h(x_2, x_3, b_2, b_3) = \left(\frac{\mathrm{i}\pi}{2}\right)^2 H_0^1(M_B F b_2) S_t(x_3) \times [\theta(b_3 - b_2) J_0(b_2 M_B D_a) H_0^1(b_3 M_B D_a) + \theta(b_2 - b_3) J_0(b_3 M_B D_a) H_0^1(b_2 M_B D_a)], \quad (A.3)$$

$$h'(x_2, x_3, b_2, b_3) = \left(\frac{\mathrm{i}\pi}{2}\right)^2 H_0^1(M_B F b_3) S_t(x_3) \times [\theta(b_3 - b_2) J_0(b_2 M_B D_b) H_0^1(b_3 M_B D_b) + \theta(b_2 - b_3) J_0(b_3 M_B D_b) H_0^1(b_2 M_B D_b)], \quad (A.4)$$

with the notation F and D standing for

$$F = \sqrt{[(1-x_2)(1-r) + x_3 r][x_3(1-r) + (1-x_2 r)]},$$
$$D_a = \sqrt{[x_3 + r(1-x_3)][1 - r(1-x_3)]},$$
$$D_b = \sqrt{[1 - x_2 + r x_2](1 - r x_2)}. \quad (A.5)$$

t is the hard scale, which is chosen as

$$t_a = \max(M_B D_a, 1/b_2, 1/b_3),$$
$$t_b = \max(M_B D_b, 1/b_2, 1/b_3). \quad (A.6)$$

The Sudakov form factor is written as

$$S_\phi(t) = \exp\left[-s(x_2 P_2^+, b_2) - s((1-x_2)P_2^+, b_2) - 2\int_{1/b_2}^{t} \frac{\mathrm{d}\bar{\mu}}{\bar{\mu}} \gamma\left(\alpha_{\mathrm{s}}\left(\bar{\mu}^2\right)\right)\right], \quad (A.7)$$

with the quark anomalous dimension $\gamma = -\alpha_{\mathrm{s}}/\pi$ and $s(Q, b)$, the so-called Sudakov factor, which comes from the resummation of the double logarithms, is given as

$$s(Q, b) = \int_{1/b}^{Q} \frac{\mathrm{d}\mu'}{\mu'} \left[\left\{\frac{2}{3}(2\gamma_{\mathrm{E}} - 1 - \log 2) + C_F \log\frac{Q}{\mu'}\right\}\frac{\alpha_{\mathrm{s}}(\mu')}{\pi} + \left\{\frac{67}{9} - \frac{\pi^2}{3} - \frac{10}{27}n_f + \frac{2}{3}\beta_0 \log\frac{\gamma_{\mathrm{E}}}{2}\right\}\left(\frac{\alpha_{\mathrm{s}}(\mu')}{\pi}\right)^2 \times \log\frac{Q}{\mu'}\right]. \quad (A.8)$$

The non-factorizable amplitudes for the diagrams in Fig. 1c,d are written as

$$M_{\mathrm{L}}^{c} = -\frac{32\pi C_F M_B^2}{\sqrt{6}} \times \int [\mathrm{d}x] \int_0^\infty b_1 \mathrm{d}b_1 b_3 \mathrm{d}b_3 \phi_B(x_1) \times \{[-1 + x_2 + r(2 - 4x_2)] \times \phi(x_2)\phi(x_3) + r(1 + x_2 - x_3)\phi^t(x_2)\phi^t(x_3) + r(-1 + x_2 + x_3)\phi^t(x_2)\phi^s(x_3) + r(1 - x_2 - x_3)\phi^s(x_2)\phi^t(x_3) + r(3 - x_2 + x_3)\phi^s(x_2)\phi^s(x_3)\}\alpha_{\mathrm{s}}(t) \times [C_4 + C_6 - C_8/2 - C_{10}/2] \times h_n(x_1, x_2, x_3, b_1, b_3) S(t), \quad (A.9)$$

$$M_{\mathrm{L}}^{d} = -\frac{32\pi C_F M_B^2}{\sqrt{6}} \times \int_0^1 [\mathrm{d}x] \int_0^\infty b_1 \mathrm{d}b_1 b_3 \mathrm{d}b_3 \phi_B(x_1) \times \{[x_3 - 4x_3 r]\phi(x_2)\phi(x_3) + r(1 - x_2 + x_3)\phi^t(x_2)\phi^t(x_3) - r(1 - x_2 - x_3)\phi^t(x_2)\phi^s(x_3) + r(1 - x_2 - x_3)\phi^s(x_2)\phi^t(x_3) + r(-1 + x_2 - x_3)\phi^s(x_2)\phi^s(x_3)\}\alpha_{\mathrm{s}}(t) \quad (A.10)$$
$$\times [C_4 + C_6 - C_8/2 - C_{10}/2](t) \times h_n'(x_1, x_2, x_3, b_1, b_3) S(t),$$

$$\mathcal{M}_{\rm N}^{c} = -\frac{32\pi C_F M_B^2}{\sqrt{6}} \times \int [{\rm d}x] \int_0^\infty b_1 {\rm d}b_1 b_3 {\rm d}b_3 \phi_B(x_1) r \times[-2\phi^v(x_2)\phi^v(x_3) + \phi^{\rm T}(x_2)\phi^{\rm T}(x_3)(1+x_2-x_3) -2\phi^a(x_2)\phi^a(x_3)]\alpha_{\rm s}(t) \times [C_4 + C_6 - C_8/2 - C_{10}/2](t) \times h_n(x_1,x_2,x_3,b_1,b_3)S(t), \quad (A.11)$$

$$\mathcal{M}_{\rm N}^{d} = -\frac{32\pi C_F M_B^2}{\sqrt{6}} \times \int [{\rm d}x] \int_0^\infty b_1 {\rm d}b_1 b_3 {\rm d}b_3 \phi_B(x_1) r \times\phi^{\rm T}(x_2)\phi^{\rm T}(x_3)(1-x_2+x_3)\alpha_{\rm s}(t) \times[C_4 + C_6 - C_8/2 - C_{10}/2](t) \times h'_n(x_1,x_2,x_3,b_1,b_3)S(t), \quad (A.12)$$

$$\mathcal{M}_{\rm T}^{c} = \frac{64\pi C_F M_B^2}{\sqrt{6}} \times \int [{\rm d}x] \int_0^\infty b_1 {\rm d}b_1 b_3 {\rm d}b_3 \phi_B(x_1) r[2\phi^v(x_2)\phi^a(x_3) +\phi^{\rm T}(x_2)\phi^{\rm T}(x_3)(1-x_2-x_3) + 2\phi^a(x_2)\phi^v(x_3)] \times\alpha_{\rm s}(t)[C_4 - C_6 + C_8/2 - C_{10}/2](t) \quad (A.13) \times h_n(x_1,x_2,x_3,b_1,b_3)S(t),$$

$$\mathcal{M}_{\rm T}^{d} = -\frac{64\pi C_F M_B^2}{\sqrt{6}} \times \int [{\rm d}x] \int_0^\infty b_1 {\rm d}b_1 b_3 {\rm d}b_3 \phi_B(x_1) r \times\phi^{\rm T}(x_2)\phi^{\rm T}(x_3)(1-x_2-x_3) \times\alpha_{\rm s}(t)[C_4 - C_6 + C_8/2 - C_{10}/2](t) \times h'_n(x_1,x_2,x_3,b_1,b_3)S(t). \quad (A.14)$$

The h functions are defined as

$$h_n(x_1,x_2,x_3,b_1,b_3) = \frac{{\rm i}\pi}{2}\left[\theta(b_3-b_1)J_0(b_1M_BF)H_0^1(b_3M_BF) \quad (A.15) + \theta(b_1-b_3)J_0(b_3M_BF)H_0^1(b_1M_BF)\right]K_0(M_BD_cb_1),$$

$$h'_n(x_1,x_2,x_3,b_1,b_3) = \frac{{\rm i}\pi}{2}\left[\theta(b_3-b_1)J_0(b_1M_BF)H_0^1(b_3M_BF) +\theta(b_1-b_3)\,J_0(b_3M_BF)H_0^1(b_1M_BF)\right] \times\begin{cases} \frac{{\rm i}\pi}{2}H_0^{(1)}\left(\sqrt{-X}b_1\right), & X<0, \\ K_0\left(\sqrt{X}b_1\right), & X>0, \end{cases} \quad (A.16)$$

with the notation

$$\int [{\rm d}x] = \int_0^1 {\rm d}x_1 \int_0^1 {\rm d}x_2 \int_0^1 {\rm d}x_3, \quad (A.17)$$

$$D_c = (1-[x_2-x_1+r(1-x_2-x_3)] \times[1-x_3-r(1-x_2-x_3)])^{1/2}, \quad (A.18)$$

$$X = [x_2+x_1-1+r(1-x_2-x_3)] \times[x_3+r(1-x_2-x_3)], \quad (A.19)$$

and the hard scale t is

$$t_c = \max(M_BF, M_BD_c, 1/b_1, 1/b_3), \quad (A.20)$$

$$t_d = \max\left(M_BF, M_B\sqrt{|X|}, 1/b_1, 1/b_3\right). \quad (A.21)$$

The Sudakov form factor is $S(t) = S_B(t)S_\phi^2(t)$, with

$$S_B(t) = s(x_1P_1^+, b_1) + 2\int_{1/b_1}^t \frac{{\rm d}\bar\mu}{\bar\mu}\gamma\left(\alpha_{\rm s}\left(\bar\mu^2\right)\right). \quad (A.22)$$

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